Appendix 3

Formulae for control charts

The constants in the following formulae may be found in Appendix 2

Charts based on variables (measurements)

Individuals chart

$$\text{UCL} = \bar{x} + 2.66\overline{MR}$$

$$\text{LCL} = \bar{x} - 2.66\overline{MR}$$ These formulae apply with moving ranges of length 2.

$$\text{CL} = \bar{x}$$

Moving range chart

$$\text{UCL} = 3.267\overline{MR}$$

LCL is effectively zero The formula for UCL applies with moving ranges of length two.

$$\text{CL} = \overline{MR}$$

Mean or Xbar chart (range based)

$$\text{UCL} = \bar{\bar{x}} + A_2\bar{R}$$

$$\text{LCL} = \bar{\bar{x}} - A_2\bar{R}$$

$$\text{CL} = \bar{\bar{x}}$$

Six Sigma Quality Improvement with Minitab, Second Edition. G. Robin Henderson.

Range chart

$$\begin{aligned} \text{UCL} &= D_4\bar{R} \\ \text{LCL} &= D_3\bar{R} \\ \text{CL} &= \bar{R} \end{aligned}$$

Mean or Xbar chart (standard deviation based)

$$\begin{aligned} \text{UCL} &= \bar{\bar{x}} + A_3\hat{\sigma} \\ \text{LCL} &= \bar{\bar{x}} - A_3\hat{\sigma} \\ \text{CL} &= \bar{\bar{x}} \end{aligned}$$

Standard deviation chart

$$\begin{aligned} \text{UCL} &= B_4\hat{\sigma} \\ \text{LCL} &= B_3\hat{\sigma} \\ \text{CL} &= \bar{s} \end{aligned}$$

EWMA chart

$$\text{UCL} = \bar{\bar{x}} + 3\hat{\sigma}\sqrt{\frac{\alpha}{2-\alpha}} \quad \text{in the long term}$$

$$\text{CL} = \bar{\bar{x}}$$

$$\text{LCL} = \bar{\bar{x}} - 3\hat{\sigma}\sqrt{\frac{\alpha}{2-\alpha}} \quad \text{in the long term}$$

where $\hat{\sigma}$ denotes an estimate of the process standard deviation and α denotes the smoothing constant.

Charts based on attributes (counts)

Proportion nonconforming (P) chart

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\text{CL} = \bar{p}$$

Number nonconforming (NP) chart

$$\text{UCL} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$\text{LCL} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$\text{CL} = n\bar{p}$$

Count of nonconformities (P) chart

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}}$$

$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}}$$

$$\text{CL} = \bar{c}$$

Mean number of nonconformities per unit (U) chart

$$\text{UCL} = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$$

$$\text{LCL} = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}}$$

$$\text{CL} = \bar{u}$$